Registration No.: | | | | | | | | | | |

Total Number of Pages: 02

Course: M.Sc.I
Sub_Code: FMCC501

5th Semester Regular Examination: 2024-25
SUBJECT: Advance Calculus
BRANCH(S): M.Sc.I(MC)
Time: 3 Hours
Max Marks: 70
Q.Code: R211
Answer Question No.1 (Part-I) which is compulsory, any five from rest (Part-II)
The figures in the right-hand margin indicate marks.

Part-I

Q1 **Answer the following questions:** **(2 x 10)**

a) What is the Z-transform of Unit impulse sequence $\delta(n)$?

b) Compute $\Gamma\left(\frac{9}{2}\right)$.

c) Define Rodrigue's formula. Using it find $P_1(x)$.

d) Find the surface of a minimum area, stretched over a given closed space curve C, enclosing the domain D in xy −plane.

e) Test for an extremum of the functional

$$I[y(x)] = \int_0^1 (xy + y^2 - 2y^2y')dx,\ y(0) = 1, y(1) = 2.$$

f) Find the geodesic curves on a surface on which a line element ds is of the form

$$(ds)^2 = [\phi(x) + \psi(y)][(dx)^2 + (dy)^2].$$

g) Find the first eigenvalue of the problem

$$y''(x) + \lambda(1 + x^2)y = 0, y(-1) = y(1) = 0.$$

h) Find the solution of Volterra integral equation $y(x) = 1 + x + \int_0^x (x - t)y(t)dt$.

i) Define separable kernel and resolvent kernel of Fredholm integral equation.

j) Show that the integral equation $g(x) = \lambda\int_0^1 (3x - 2)t\ g(t)\ dt$ has no characteristic number.

Part-II

Long Answer Type Questions (Answer Any five)

Q2 **a)** Show that $P_4(x)$ is solution of Legender polynomial. **(5+5)**

b) Establish the relation between Beta and Gamma functions.

Q3 **a)** Find the inverse Fourier sine transform of $\frac{1}{s}e^{-as}$. **(5+5)**

b) Find $J_{\frac{1}{2}}(x)$ and $J_{-\frac{1}{2}}(x)$ and check whether they are linearly dependent or not.

Q4 a) Find the extremum of the function **(5+5)**

$$I[y(x)] = \int_{x_0}^{x_1} \frac{(1+y')^{\frac{1}{2}}}{x} dx.$$

b) Find the inverse Z-transform of $\frac{4z^3-2z}{z^3-5z^2+8z-4}$.

Q5 a) Minimize the integral **(5+5)**

$$I[y] = \int_{-l}^{l} \frac{y'(s)}{x-s}\, ds) y(x) dx$$

subject to the constraint

$$J[y] = \int_{-l}^{l} y(x)\, dx = S = \text{constant}.$$

And the boundary conditions $y(l) = y(-l) = 0$.

b) Find the shortest distance between the parabola $y = x^2$ and the straight line $\mathrm{x} - \mathrm{y} = 5$.

Q6 a) Find the extremal with corner points of the functional **(5+5)**

$$I[y(x)] = \int_{x_1}^{x_2} y'^2 (1-y')^2 dx.$$

b) Determine the shape of solid of revolution moving in a flow of gas dynamic.

Q7 a) Show that $y(x) = cos2x$ is a solution of the integral equation **(5+5)**

$$y(x) = cosx + 3\int_0^{\pi} k(x,t)y(t)\, dt, \text{ where } k(x,t) = sinx\ cost, 0 \le x \le t$$
$$\text{and } \quad k(x,t) = cosx\ sint, t \le x \le \pi.$$

b) Determine the resolvent kernel for Fredholm integral equation having kernel $k(x,t) = e^{x+t}, a = 0, b = 1$.

Q8 a) Solve the integral equation $y(x) = \frac{5x}{6} + \frac{1}{2}\int_0^2 xt\, y(t)\, dt$ using Resolvent kernel. **(5+5)**

b) Find the eigenvalue and eigenfunction of $g(x) = \lambda \int_0^1 e^{\{x+t\}}\, g(t)\, dt$.